Project Proposal deadline: tonight, 11:59pm

Course Notes: <u>https://snap-stanford.github.io/cs224w-notes/</u> Help us write the course notes – we will give **generous bonuses!**

Graph Neural Networks

CS224W: Machine Learning with Graphs Jure Leskovec, Stanford University http://cs224w.stanford.edu



Node Embeddings

 Intuition: Map nodes to *d*-dimensional embeddings such that similar nodes in the graph are embedded close together



How to <u>learn</u> mapping function *f*?

 Goal: Map nodes so that similarity in the embedding space (e.g., dot product) approximates similarity (e.g., proximity) in the network



Node Embeddings



Two Key Components

• Encoder: Map a node to a low-dimensional vector: $TENC(w) = \pi^{-1}$

 $\mathrm{ENC}(v) = \mathbf{z}_v$ node in the input graph

 Similarity function defines how relationships in the input network map to relationships in the embedding space:

similarity
$$(u, v) \approx \mathbf{z}_{v}^{\top} \mathbf{z}_{u}$$

Similarity of *u* and *v* in the network

dot product between node embeddings

From "Shallow" to "Deep"

So far we have focused on "shallow" encoders, i.e. embedding lookups:



Shallow Encoders (Lec. 09: 10/23)

Shallow encoders:

- One-layer of data transformation
- A single hidden layer maps node u to embedding z_u via function f(), e.g. $z_u = f(z_v, v \in N_R(u))$



Shallow Encoders (Lec. 09: 10/23)

- Limitations of shallow embedding methods:
 - O(|V|) parameters are needed:
 - No sharing of parameters between nodes
 - Every node has its own unique embedding
 - Inherently "transductive":
 - Cannot generate embeddings for nodes that are not seen during training
 - Do not incorporate node features:
 - Many graphs have features that we can and should leverage

Today: Deep Graph Encoders

Today: We will now discuss deep methods based on graph neural networks:

$$\operatorname{ENC}(v) =$$

multiple layers of non-linear transformations of graph structure

 Note: All these deep encoders can be combined with node similarity functions defined in the last lecture

Deep Graph Encoders



Modern ML Toolbox



Modern deep learning toolbox is designed for simple sequences & grids

Why is it Hard?

But networks are far more complex!

 Arbitrary size and complex topological structure (i.e., no spatial locality like grids)



- No fixed node ordering or reference point
- Often dynamic and have multimodal features

Idea: Convolutional Networks

CNN on an image:



Goal is to generalize convolutions beyond simple lattices Leverage node features/attributes (e.g., text, images)

From Images to Graphs

Single CNN layer with 3x3 filter:



Transform information at the neighbors and combine it:

- Transform "messages" h_i from neighbors: $W_i h_i$
- Add them up: $\sum_i W_i h_i$

Real-World Graphs

But what if your graphs look like this?



Examples:

Biological networks, Medical networks, Social networks, Information networks, Knowledge graphs, Communication networks, Web graph, ...

A Naïve Approach

Join adjacency matrix and featuresFeed them into a deep neural net:



- Issues with this idea:
 - O(N) parameters
 - Not applicable to graphs of different sizes
 - Not invariant to node ordering

Outline of Today's Lecture

1. Basics of deep learning for graphs $_{\odot}$

2. Graph Convolutional Networks

3. Graph Attention Networks (GAT)

4. Practical tips and demos

Basics of Deep Learning for Graphs

Content

Local network neighborhoods:

- Describe aggregation strategies
- Define computation graphs

Stacking multiple layers:

- Describe the model, parameters, training
- How to fit the model?
- Simple example for unsupervised and supervised training



- Assume we have a graph G:
 - V is the vertex set
 - *A* is the **adjacency matrix** (assume binary)
 - $X \in \mathbb{R}^{m \times |V|}$ is a matrix of **node features**
 - Node features:
 - Social networks: User profile, User image
 - Biological networks: Gene expression profiles, gene functional information
 - No features:
 - Indicator vectors (one-hot encoding of a node)
 - Vector of constant 1: [1, 1, ..., 1]

[Kipf and Welling, ICLR 2017] Graph Convolutional Networks

Idea: Node's neighborhood defines a computation graph

 $\begin{bmatrix} x_1 \\ x_2 \\ \cdot \end{bmatrix}$



Determine node computation graph Propagate and transform information

aggregator

aggregator

Learn how to propagate information across the graph to compute node features

Idea: Aggregate Neighbors

Key idea: Generate node embeddings based on local network neighborhoods



Idea: Aggregate Neighbors

Intuition: Nodes aggregate information from their neighbors using neural networks



Idea: Aggregate Neighbors

 Intuition: Network neighborhood defines a computation graph



Deep Model: Many Layers

- Model can be of arbitrary depth:
 - Nodes have embeddings at each layer
 - Layer-0 embedding of node u is its input feature, x_u
 - Layer-K embedding gets information from nodes that are K hops away



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Neighborhood Aggregation

 Neighborhood aggregation: Key distinctions are in how different approaches aggregate information across the layers



Neighborhood Aggregation

 Basic approach: Average information from neighbors and apply a neural network



The Math: Deep Encoder

Basic approach: Average neighbor messages and apply a neural network



Training the Model



Need to define a loss function on the embeddings

Model Parameters

$$\mathbf{h}_{v}^{0} = \mathbf{x}_{v} \quad \text{(i.e., what we learn)} \\ \mathbf{h}_{v}^{k} = \sigma \left(\underbrace{\mathbf{W}_{k}}_{u \in N(v)} \underbrace{\mathbf{h}_{u}^{k-1}}_{|N(v)|} + \underbrace{\mathbf{B}_{k}}_{v} \mathbf{h}_{v}^{k-1} \right), \quad \forall k \in \{1, ..., K\} \\ \mathbf{z}_{v} = \mathbf{h}_{v}^{K}$$

We can feed these **embeddings into any loss function** and run stochastic gradient descent to train the weight parameters

Equivalently rewritten in vector form:

$$\mathbf{H}^{(l+1)} = \sigma \left(\mathbf{H}^{(l)} \mathbf{W}_{0}^{(l)} + \tilde{\mathbf{A}} \mathbf{H}^{(l)} \mathbf{W}_{1}^{(l)} \right)$$

with $\tilde{\mathbf{A}} = \mathbf{D}^{-\frac{1}{2}} \mathbf{A} \mathbf{D}^{-\frac{1}{2}}$

 $A = D^{-2}AD$

 $\mathbf{H}^{(l)} = [\mathbf{h}_{1}^{(l)^{T}}, \dots, \mathbf{h}_{N}^{(l)^{T}}]^{T}$

Unsupervised Training

- Train in an unsupervised manner:
 - Use only the graph structure
 - "Similar" nodes have similar embeddings
- Unsupervised loss function can be anything from the last section, e.g., a loss based on
 - Random walks (node2vec, DeepWalk, struc2vec)
 - Graph factorization
 - Node proximity in the graph

Supervised Training

Directly train the model for a supervised task (e.g., node classification)



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Supervised Training

Directly train the model for a supervised task (e.g., node classification)



Model Design: Overview



Model Design: Overview



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Model Design: Overview



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Inductive Capability

- The same aggregation parameters are shared for all nodes:
 - The number of model parameters is sublinear in |V| and we can generalize to unseen nodes!



Inductive Capability: <u>New Graphs</u>



Inductive node embedding \longrightarrow Generalize to entirely unseen graphs

E.g., train on protein interaction graph from model organism A and generate embeddings on newly collected data about organism B

Inductive Capability: <u>New Nodes</u>



- Many application settings constantly encounter previously unseen nodes:
- E.g., Reddit, YouTube, Google Scholar
 Need to generate new embeddings "on the fly"

Summary So Far

- Recap: Generate node embeddings by aggregating neighborhood information
 - We saw a basic variant of this idea
 - Key distinctions are in how different approaches aggregate information across the layers
- Next: Describe GraphSAGE graph neural network architecture

Outline of Today's Lecture

1. Basics of deep learning for graphs

2. Graph Convolutional Networks

3. Graph Attention Networks (GAT)

4. Practical tips and demos



Graph Convolutional Networks and GraphSAGE

GraphSAGE Idea

So far we have aggregated the neighbor messages by taking their (weighted) average Can we do better?



GraphSAGE Idea



Apply L2 normalization for each node embedding at every layer

Neighborhood Aggregation

Simple neighborhood aggregation:

$$\mathbf{h}_{v}^{k} = \sigma \left(\mathbf{W}_{k} \sum_{u \in N(v)} \frac{\mathbf{h}_{u}^{k-1}}{|N(v)|} + \mathbf{B}_{k} \mathbf{h}_{v}^{k-1} \right)$$

• GraphSAGE: $\mathbf{h}_{v}^{k} = \sigma\left(\left[\mathbf{W}_{k} \cdot \operatorname{AGG}\left(\left\{\mathbf{h}_{u}^{k-1}, \forall u \in N(v)\right\}\right), \mathbf{B}_{k}\mathbf{h}_{v}^{k-1}\right]\right)$ Generalized aggregation

Neighbor Aggregation: Variants

Mean: Take a weighted average of neighbors

$$AGG = \sum_{u \in N(v)} \frac{\mathbf{h}_u^{k-1}}{|N(v)|}$$

- Pool: Transform neighbor vectors and apply symmetric vector function Element-wise mean/max $AGG = \gamma (\{\mathbf{Qh}_{u}^{k-1}, \forall u \in N(v)\})$
- LSTM: Apply LSTM to reshuffled of neighbors $AGG = LSTM([\mathbf{h}_{u}^{k-1}, \forall u \in \pi(N(v))])$

Recap: GCN, GraphSAGE

Key idea: Generate node embeddings based on local neighborhoods

- Nodes aggregate "messages" from their neighbors using neural networks
- Graph convolutional networks:
 - Basic variant: Average neighborhood information and stack neural networks
- GraphSAGE:
 - Generalized neighborhood aggregation



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Efficient Implementation

 Many aggregations can be performed efficiently by (sparse) matrix operations

• Let $H^{k-1} = [\mathbf{h}_1^{k-1} \dots \mathbf{h}_n^{k-1}]$



Another example: GCN (Kipf et al. 2017)

$$H^k = D^{-1/2} A D^{1/2} H^{k-1}$$

More on Graph Neural Networks

Tutorials and overviews:

- Relational inductive biases and graph networks (Battaglia et al., 2018)
- Representation learning on graphs: Methods and applications (Hamilton et al., 2017)

Attention-based neighborhood aggregation:

Graph attention networks (Hoshen, 2017; Velickovic et al., 2018; Liu et al., 2018)

Embedding entire graphs:

- Graph neural nets with edge embeddings (Battaglia et al., 2016; Gilmer et. al., 2017)
- Embedding entire graphs (Duvenaud et al., 2015; Dai et al., 2016; Li et al., 2018) and graph pooling (Ying et al., 2018, Zhang et al., 2018)
- Graph generation and relational inference (You et al., 2018; Kipf et al., 2018)
- How powerful are graph neural networks(Xu et al., 2017)

Embedding nodes:

- Varying neighborhood: Jumping knowledge networks (Xu et al., 2018), GeniePath (Liu et al., 2018)
- Position-aware GNN (You et al. 2019)

Spectral approaches to graph neural networks:

- Spectral graph CNN & ChebNet (Bruna et al., 2015; Defferrard et al., 2016)
- Geometric deep learning (Bronstein et al., 2017; Monti et al., 2017)

Other GNN techniques:

- Pre-training Graph Neural Networks (Hu et al., 2019)
- GNNExplainer: Generating Explanations for Graph Neural Networks (Ying et al., 2019)

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Graph Attention Networks

Simple Neighborhood Aggregation

Recap: Simple neighborhood aggregation:

$$\mathbf{h}_{v}^{k} = \sigma \left(\mathbf{W}_{k} \sum_{u \in N(v)} \frac{\mathbf{h}_{u}^{k-1}}{|N(v)|} + \mathbf{B}_{k} \mathbf{h}_{v}^{k-1} \right)$$

- Graph convolutional operator:
 - Aggregates messages across neighborhoods, N(v)
 - $\alpha_{vu} = 1/|N(v)|$ is the weighting factor (importance) of node u's message to node v
 - $\Rightarrow \alpha_{vu}$ is defined **explicitly** based on the structural properties of the graph
 - ⇒ All neighbors $u \in N(v)$ are equally important to node v

[Velickovic et al., ICLR 2018; Vaswani et al., NIPS 2017]

Graph Attention Networks

Can we do better than simple neighborhood aggregation?

Can we let weighting factors α_{vu} to be implicitly defined?

- Goal: Specify arbitrary importances to different neighbors of each node in the graph
- Idea: Compute embedding h^k_v of each node in the graph following an attention strategy:
 - Nodes attend over their neighborhoods' message
 - Implicitly specifying different weights to different nodes in a neighborhood

Attention Mechanism (1)

- Let \(\alpha_{\nuu}\) be computed as a byproduct of an attention mechanism \(\alpha\):
 - Let a compute attention coefficients e_{vu} across pairs of nodes u, v based on their messages:

$$e_{vu} = a(\boldsymbol{W}_k \boldsymbol{h}_u^{k-1}, \boldsymbol{W}_k \boldsymbol{h}_v^{k-1})$$

• e_{vu} indicates the importance of node u's message to node v

 Normalize coefficients using the softmax function in order to be comparable across different neighborhoods:

$$\alpha_{vu} = \frac{\exp(e_{vu})}{\sum_{k \in N(v)} \exp(e_{vk})}$$
$$h_v^k = \sigma(\sum_{u \in N(v)} \alpha_{vu} W_k h_u^{k-1})$$
Next: What is the form of attention mechanism *a*

Attention Mechanism (2)

- Attention mechanism a:
 - The approach is agnostic to the choice of a
 - E.g., use a simple single-layer neural network
 - a can have parameters, which need to be estimates
 - Parameters of a are trained jointly:
 - Learn the parameters together with weight matrices (i.e., other parameter of the neural net) in an end-to-end fashion
- Multi-head attention: Stabilize the learning process of attention mechanism [Velickovic et al., ICLR 2018]:
 - Attention operations in a given layer are independently replicated R times (each replica with different parameters)
 - Outputs are aggregated (by concatenating or adding)

Properties of Attentional Mechanism

 Key benefit: Allows for (implicitly) specifying different importance values (α_{vu}) to different neighbors

Computationally efficient:

- Computation of attentional coefficients can be parallelized across all edges of the graph
- Aggregation may be parallelized across all nodes

Storage efficient:

- Sparse matrix operations do not require more than O(V+E) entries to be stored
- **Fixed** number of parameters, irrespective of graph size
- Trivially localized:
 - Only attends over local network neighborhoods

Inductive capability:

- It is a shared *edge-wise* mechanism
- It does not depend on the global graph structure

GAT Example: Cora Citation Net

Method	Cora
MLP	55.1%
ManiReg (Belkin et al., 2006)	59.5%
SemiEmb (Weston et al., 2012)	59.0%
LP (Zhu et al., 2003)	68.0%
DeepWalk (Perozzi et al., 2014)	67.2%
ICA (Lu & Getoor, 2003)	75.1%
Planetoid (Yang et al., 2016)	75.7%
Chebyshev (Defferrard et al., 2016)	81.2%
GCN (Kipf & Welling, 2017)	81.5%
GAT	83.3%
improvement w.r.t GCN	1.8%
Attention mechanism can be us	ed with
many different graph neural ne	twork
models	
models	
In many cases, attention leads t	to.
norformance gains	

- t-SNE plot of GAT-based node embeddings:
 - Node color: 7 publication classes
 - Edge thickness: Normalized attention coefficients between nodes *i* and *j*, across eight attention heads, $\sum_{k} (\alpha_{ij}^{k} + \alpha_{ji}^{k})$

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Example Application

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Application: Pinterest



Christing saved to Kitchen

10/17/10





Blue accents 219 Pins



Vintage kitchen 377 Pins



300M users4+B pins, 2+B boards

Application: Pinterest

Human curated collection of pins



Very ape blue structured coat Nitty Gritty



Hans Wegner chair

oom and Board Promoted by Room & Board



This is just a beautiful #14 image for thoughts. Yay or nay, your choice.

Annie Teng Plantation

Pin: A visual bookmark someone has saved from the internet to a board they've created.Pin: Image, text, link



Board: A collection of ideas (pins having something in common)

Pinterest Graph

Graph: 2B pins, 1B boards, 20B edges

 Graph is dynamic: Need to apply to new nodes without model retraining

Rich node features: Content, images

PinSage: Overview

- PinSage graph convolutional network:
 - Goal: Generate embeddings for nodes (e.g., Pins/images) in a web-scale Pinterest graph containing billions of objects
 - Key Idea: Borrow information from nearby nodes
 - E.g., bed rail Pin might look like a garden fence, but gates and beds are rarely adjacent in the graph





- Pin embeddings are essential to various tasks like recommendation of Pins, classification, clustering, ranking
 - Services like "Related Pins", "Search", "Shopping", "Ads"

Embedding Nodes



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Task Overview

Task: Recommend related pins to users



Task: Learn nodeembeddings z_i suchthat $d(z_{cake1}, z_{cake2})$

 $< d(z_{cake1}, z_{sweater})$

Challenges:

- Massive size: 3 billion nodes, 20 billion edges
- Heterogeneous data: Rich image and text features

Goal: Identify target pin among 3B pins

- Issue: Need to learn with resolution of 100 vs. 3B
- Idea: Use harder and harder negative samples
- Include more and more hard negative samples for each epoch



Source pin



Positive





Easy negative Hard negative

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PinSAGE Efficiency

- How to scale the training as well as inference of node embeddings to graphs with billions of nodes and tens of billions of edges?
 - 10,000X larger dataset than any previous graph neural network application
- Key innovations:
 - Sub-sample neighborhoods for efficient GPU batching
 - Producer-consumer CPU-GPU training pipeline
 - Curriculum learning for negative samples
 - MapReduce for efficient inference

PinSage: Key Innovations (1)

Three key innovations:

1. On-the-fly graph convolutions

- Sample the neighborhood around a node and dynamically construct a computation graph
- Perform a localized graph convolution around a particular node
- Does not need the entire graph during training



PinSage: Key Innovations (2)

Three key innovations:

- **1.** On-the-fly graph convolutions
- 2. Constructing convolutions via random walks
 - Performing convolutions on full neighborhoods is infeasible:
 - How to select the set of neighbors of a node to convolve over?
 - Importance pooling: Define importance-based neighborhoods by simulating random walks and selecting the neighbors with the highest visit counts

3. Efficient MapReduce inference

- Bottom-up aggregation of node embeddings lends itself to MapReduce
 - Decompose each aggregation step across all nodes into three operations in MapReduce, i.e., *map*, *join*, and *reduce*

PinSage: Experiments

Baselines:

- Visual: Nearest neighbors of CNN visual embeddings for recommendations
- Annotation: Nearest neighbors in terms of Word2vec embeddings
- Combined: Concatenate embeddings:
 - Uses exact same data and loss function as PinSage

PinSage gives 150% improvement in hit rate and 60% improvement in MRR over the best baseline

Method	Hit-rate	MRR
Visual	17%	0.23
Annotation	14%	0.19
Combined	27%	0.37
max-pooling	39%	0.37
mean-pooling	41%	0.51
mean-pooling-xent	29%	0.35
mean-pooling-hard	46%	0.56
PinSage	67 %	0.59

Example Pin Recommendations



Pixie is a purely graph-based method that uses biased random walks to generate ranking scores by simulating random walks starting at query Pin. Items with top scores are retrieved as recommendations [Eksombatchai et al., 2018]

PinSAGE Recommendations



Query



PinSAGE


PinSAGE Recommendations





Query

General Tips and Practical Demos

General Tips

Data preprocessing is important:

- Use renormalization tricks
- Variance-scaled initialization
- Network data whitening
- ADAM optimizer:
 - ADAM naturally takes care of decaying the learning rate
- ReLU activation function often works really well
- No activation function at your output layer:
 - Easy mistake if you build layers with a shared function
- Include bias term in every layer
- GCN layer of size 64 or 128 is already plenty

Debugging Deep Networks

Debug?!:

Loss/accuracy not converging during training
Important for model development:

Overfit on training data:

- Accuracy should be essentially 100% or error close to 0
- If neural network cannot overfit a single data point, something is wrong
- Scrutinize your loss function!
- Scrutinize your visualizations!

Demo: Human Disease Network



Demo: Protein Interaction Prediction



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